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Special Brief Note

Vortex-induced vibration of a cylinder with two degrees of freedom

N. Jauvtis, C.H.K. Williamson*

Mechanical and Aerospace Engineering, Cornell University, 252 Upson Hall, Ithaca, NY 14853, USA

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Abstract

In this work, we study the response of an elastically mounted cylinder, which is free to move in two degrees of freedom in a fluid flow, and which has low mass and damping. There has been a great deal of work concerned with bodies restrained to move in the direction transverse to the free stream, but very few studies which comprise motion in both the transverse (Y) and in-line (X) directions. In such cases, it has generally been assumed that in-line response would dramatically change the character of the wake vortex dynamics as well as the transverse body response. We find in the present work that, surprisingly, the freedom to move in two directions has very little effect on the transverse response, the modes of vibration, or the vortex wake dynamics (for a body of similar low mass ratio (relative density) in the range $m^* = 5-25$). For low values of normalised velocity ($U^* \sim 2-4$) below the classical synchronisation regime for transverse response, we find two in-line vibration modes, which are associated with symmetric and antisymmetric vortex wake modes, corresponding well with the modes discovered by Wooton et al. and by King for a flexible cantilever. Coupled with a parallel effort by D.O. Rockwell's group at Lehigh, these experiments form the first such studies in which both the oscillating mass and the natural frequency are precisely the same in the X and Y directions. A principal conclusion from this investigation is that it demonstrates the validity, for bodies in two degrees of freedom, of employing the existing comprehensive results for bodies restrained to vibrate only in the transverse Y-direction, even down to low mass ratios of $m^* = 5$.

1. Introduction

Vortex-induced vibration of structures is of practical interest in many branches of engineering, for example in aeroelastic applications where the fluid is air, yielding mass ratios m^* of order 100 (m^* = mass of oscillating structure/displaced fluid mass), or in hydroelastic applications in water, where m^* is of order 1 or 10. In most practical cases, cylindrical structures (such as riser tubes or heat exchangers; to name two examples) have a mass ratio, which is the same in both the streamwise (X) and transverse (Y) directions, and the same natural frequencies in these two directions. Contrasting with previous studies, we therefore focus on a design which ensures similar mass and similar frequencies in these two directions, and we introduce the resulting pendulum apparatus later in Section 2.

The problem of vortex-induced vibration of cylinders free to respond transverse to the fluid flow has been well studied, and several reviews discuss this problem [see Sarpkaya (1979); Bearman (1984); Naudascher and Rockwell (1993); Sumer and Fredsoe (1997), for example]. The work of Feng (1968) at high mass ratios, $m^* = 320$, demonstrates that the resonance of a body, when the oscillation frequency coincides with the vortex formation frequency, will occur over a regime of normalised velocity U^* (where $U^* = U/f_N D$; f_N =natural frequency; D =diameter) such that

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*Corresponding author.

E-mail address: cw26@cornell.edu (C.H.K. Williamson).

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$U^* \sim 5-8$. Two response amplitude branches are found, which are shown by Brika and Laneville (1993) and Govardhan and Williamson (2000) to be due to two modes of vortex formation, as follows. For the "Initial" branch of response, the vortex wake comprises a "2S" mode, representing two *single vortices* formed per cycle. The "lower" branch comprises the "2P" mode, whereby two *vortex pairs* are formed per cycle (as originally defined in Williamson and Roshko (1988) from their forced vibration study). However, at low mass and damping (m^* typically of the order 5–10), and higher amplitudes of response, three response branches are found to exist (Khalak and Williamson, 1996, 1999; Govardhan and Williamson, 2000), namely the Initial branch (with a 2S vortex wake mode), the Upper branch (with a 2P mode), and the Lower branch (with a 2P mode). These modes and responses are found for strictly transverse vibration in a fluid flow, and the significant question arises: *To what extent are these transverse response modes and amplitudes influenced by the body's freedom to respond in the streamwise direction*?

Two studies have been carried out for systems enabling XY motion of a cylinder. In the case of Moe and Wu (1990), the mass ratios in the X and Y directions are quite different, and also the natural frequencies are set in the ratio $f_X/f_Y = 2.18$. Under these special conditions, they find a broad regime of velocity U^* over which resonant amplitudes are found (with transverse normalised amplitude close to $A_Y^* = A/D = 1$), but no evidence of distinct response branches. For Y motion, one expects a resonance when the speed of the flow is such that the vortex frequency for the non-oscillating body (f_V) is near to the structural natural frequency (f_N), which will occur when $U^* = U/f_N D \sim U/f_V D = 1/S$, where S =Strouhal number. With a Strouhal number of around 0.2, one expects resonant oscillations near a velocity, $U^* \sim 5$. For their XY experiments, Moe and Wu find that the position of maximum response shifts to rather higher values of U^*, and reaches slightly higher amplitudes, as compared with Y-motion experiments. Later experiments of Sarpkaya (1995) have concentrated on various ratios between f_X and f_Y, although he shows one set of amplitude data for XY motion where $f_X = f_Y$, indicating a slight increase in transverse amplitude and a shift to higher U^* for the peak transverse response, when compared with the Y-motion case. In this case also, the oscillating masses in each of the X and Y directions are different. He states that the results obtained in Y-only experiments "are not expected to remain valid" for XY motion, and that they "may be of limited use under certain circumstances", where the natural frequencies f_X and f_Y are quite dissimilar. However, in the present study, where the mass and natural frequencies are precisely the same in the X and Y directions, we shall see that there is a remarkable similarity in the responses between XY and Y motion.

A different approach was adopted recently by Jeon and Gharib (2001), who forced a cylinder to move in the X and Y directions, in a fluid flow, under the prescribed motions:

$$x = A_X \sin(2\omega t + \theta), \tag{1a}$$

$$y = A_Y \sin(\omega t), \tag{1b}$$

where specific phase angles, $\theta = 0°$ and $-45°$, were chosen, since they stated that "nature prefers a figure-eight-type motion". One of the most interesting results from this study appears to be the fact that even small amounts of streamwise motion (A_X/A_Y =20%) can inhibit the formation of the 2P mode of vortex formation. However, it should be mentioned that the free vibration studies of the present work, with a whole range of mass ratios chosen (including those not included here, for brevity), indicate body motions which can be quite different from a figure-of-eight motion as assumed above. In fact, at the highest amplitudes, the body trajectories are crescent-shaped (which is also the type of motion found for a tethered sphere in a flow, under some conditions; Govardhan and Williamson, 1997, 2003), and the phase has a value closer to $\theta = -90°$. Clearly, the choice of which amplitudes and phases are selected in an XY forced-vibration experiment (out of a rather large range of possibilities) will influence the resulting conclusions.

Finally, it is important to note that full-scale piles in an ocean current, and model cantilevers in the laboratory, have been found to vibrate, even leading to structural failure. It was discovered by Wooton et al. (1972) and King (1974) that these vibrations involved significant streamwise motion, although with peak amplitudes ($A_X^* \sim 0.15$) less than those found for resonant transverse vibration ($A_Y^* \sim 1.0$). As reviewed by Bearman (1984), and by Naudascher (1987), these streamwise vibrations are due to the fact that, as each vortex is shed, a fluctuating drag is generated, so that the forcing frequency is twice that for the transverse direction. The forcing induces the body to vibrate in-line with the flow, if the normalised velocity is close to $U^* \sim 1/2S$, and King (1974) showed that the wake formation in this case comprised a classical vortex street (antisymmetric) pattern. Interestingly, they also discovered a second mode of streamwise vibration, which occurred for slightly lower U^*, when the vortex wake formed symmetric pairs close to the body, giving rise to a force in phase with the velocity, and an energy transfer to body motion. These two modes have subsequently been studied using forced vibration of a cylinder, by Ongoren and Rockwell (1988). We shall find, in the present study, that an elastically mounted (otherwise rigid) cylinder in XY motion can also exhibit both of these modes, which is not unexpected.

2. Experimental details

We have constructed a hydroelastic apparatus, for particular application to very low mass and damping conditions, which operates in conjunction with the Cornell-ONR Water Channel. One may refer to Khalak and Williamson (1996) for the details concerning this water channel facility. A horizontal plate over the water channel is suspended by four cables from the roof of the laboratory, and this plate acts as a pendulum, below which is mounted a vertical cylinder that reaches down into the fluid flow of the water channel. The cylinder is thus able to move in-line and transverse to the free stream, and has the same natural frequency (typically $f_N = 0.4\,\mathrm{Hz}$) and oscillating mass ($m^* = 5-25$) in these two directions, which was an essential design requirement. We use cylinders of submerged length = 38.1 cm, and diameter = 3.81 or 5.08 cm, and we have a range of Reynolds numbers from 1000–6000. Very low values of the mass-damping parameters were used $(m^* + C_A)\zeta = 0.01$–0.10 (where C_A = ideal added mass coefficient and ζ is the structural damping coefficient). Displacement was measured using magneto-strictive (non-contact) instrumentation. Digital particle image velocimetry (DPIV) was used to determine the vorticity in a plane midway down the submerged cylinder length, and the implementation of this technique is described in detail in Govardhan and Williamson (2000). The coordinate system is defined such that the origin is where the cylinder axis intersects the free surface; x is the downstream axis, y is the transverse axis, and z is the downward vertical axis of the cylinder.

3. Response modes and vortex dynamics for a cylinder in XY motion

One of the principal interests in undertaking this research is to determine the extent to which freedom to move in-line with the flow, simultaneous with motion transverse to the flow, will modify the types of response that have been measured extensively for Y-only motions. It has been suggested in previous work that there will be dramatic changes in the response of such structures where X, as well as Y, motions are possible. Interestingly, the response, even at low values of mass ratio ($m^* = 6.9$), and low mass damping: $(m^* + C_A)\zeta = 0.0115$, is remarkably unaffected by the presence of two degrees of freedom, as shown in the response plot of Fig. 1, showing amplitude A_Y^* versus velocity U^*. For comparison, we were able to restrain the X motion in one set of data, by the use of thin very long cables attached to the carriage, and extending to the wall of the laboratory far upstream. The initial, upper and lower branches of response are found for both the cases (XY and Y-only), and there is a small (around 10%) increase in peak amplitude for the upper branch in the case of XY motions. These branches are denoted by I (initial), U (upper) and L (lower) in Fig. 1.

A set of peak amplitudes A_Y^* measured for both the upper and lower branches, as a function of the mass-damping parameter $(m^* + C_A)\zeta$, have been plotted in the "Griffin" plot of Fig. 2 (so-called after Griffin's original plots similar to this; Griffin et al., 1975; Griffin, 1980). There is only a slight difference between the peak responses measured for the XY motion as compared with the Y-only case, across the complete data set, where $(m^* + C_A)\zeta = 0.01-0.1$, and where mass ratios vary in the range, $m^* = 5.0-25.0$.

Although the in-line vibrations remained small for all the sets of experiments, there were measurable small vibrations for the upper branch response ($A_X = 0.04$), where the in-line vibrations yield a crescent-shaped XY trajectory (and a phase angle as defined from Eqs. (1a) and (1b) of around $\theta = -90^\circ$), as indicated later in Fig. 4. Streamwise vibration modes were clearly observed, and correspond with the response branches denoted SS and SA in Fig. 1. It can be seen that the first of these modes, defined as "streamwise symmetric" (SS), represents a purely in-line motion, with symmetric pairs of wake vortices being generated in each cycle of in-line motion, as shown in Fig. 3(a). This corresponds with the "first excitation" mode of a flexible cantilever, in the full-scale pile dynamics and laboratory experiments of Wooton et al. (1972) and King (1974).

In the present case of an elastically mounted rigid cylinder, one would also expect the "second excitation" mode that was found in these early studies, also to be evident here. In Fig. 1, we can see such a mode as the response branch denoted SA, or "streamwise antisymmetric", where there are now coupled motions transverse and parallel to the flow. (Comparable amplitudes are found for both modes ($A_X^* \sim 0.11-0.13$) as well as the cantilever responses ($A_X^* \sim 0.15$) studied by Wooton et al. (1972) and King (1974); see also the book by Sumer and Fredsoe (1997)). The XY trajectory is a clear figure-of-eight, where the in-line displacement dominates, and the vortex dynamics resemble the formation of a classical Karman street wake, as shown in Fig. 3(b). In this case, one major vortex is formed in each cycle of in-line motion (each half cycle of transverse motion), and the frequency of transverse motion (f) in Fig. 1 (in fact for both the SS and SS modes) is given by $f^* = f/f_N = 0.5$. Following the discussion in Section 1, this represents the condition where the range of U^* is near: $U^* \sim 1/2S \sim 2.5$. In fact, it appears that when $U^* < 1/2S$, we have the streamwise symmetric mode, and where $U^* > 1/2S$, we have the streamwise antisymmetric mode.

The modes of vortex formation for the other three response branches, where there is significant transverse motion (namely the initial, upper and lower branches) are shown in Fig. 4, and here it is evident that the vortex wake modes are

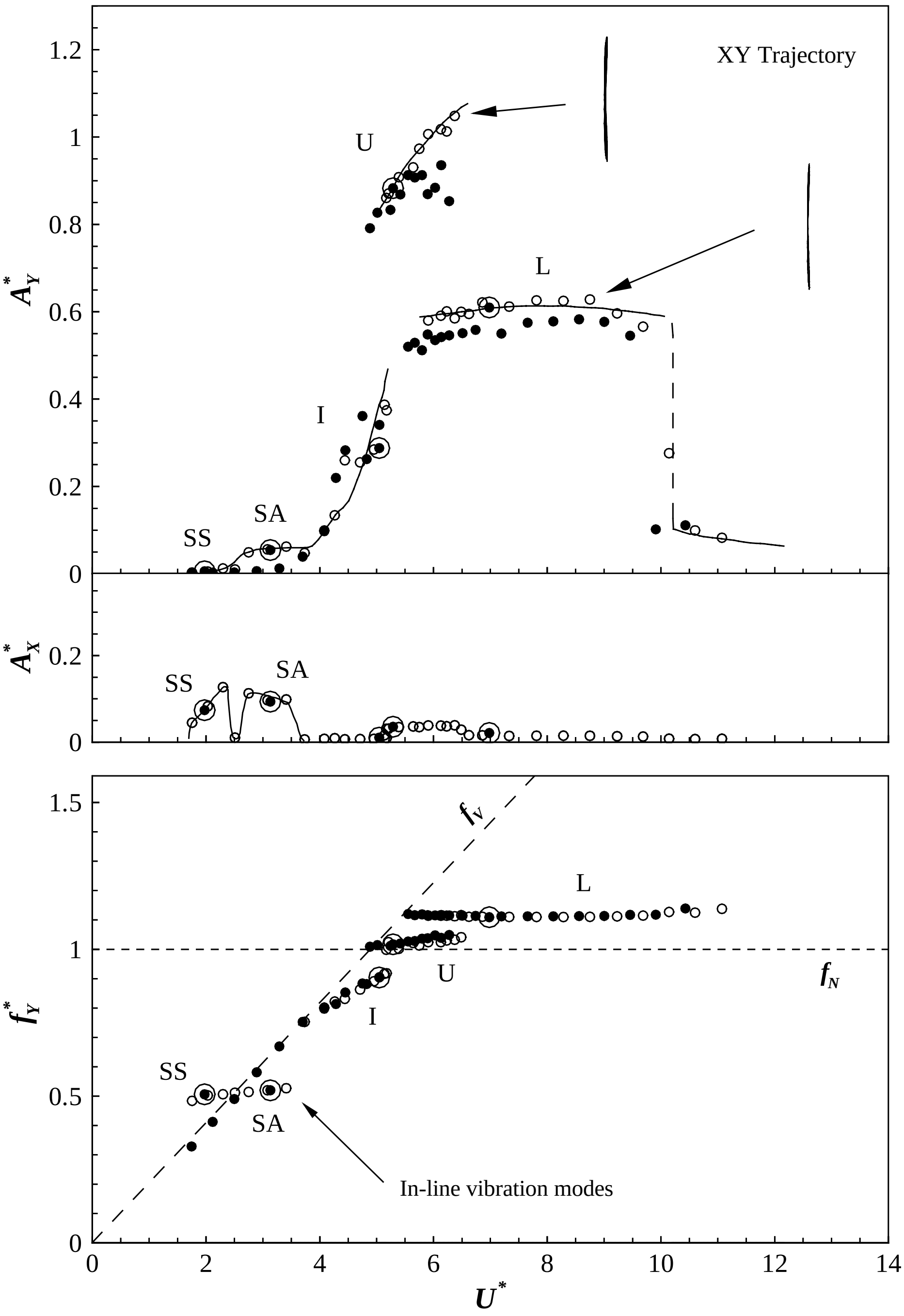

Fig. 1. Transverse and streamwise amplitudes (A_Y^* and A_X^*) and frequency (f_Y^*) response versus reduced velocity (U^*) for $m^* = 6.9$ and $(m^* + C_A)\zeta = 0.0115$. Solid symbols (●) represent Y-only data, and open symbols (○) are for XY data. The "bull's eye" symbols (⊙) indicate the points where DPIV was studied for XY motion.

the same as found for purely transverse motion (Govardhan and Wiliamson, 2000). The initial branch corresponds with a 2S vortex wake mode, while the upper and lower branches correspond with the 2P mode. The XY trajectories show that there is very little streamwise motion, and so it is not unexpected that the vortex wake modes are the same. It might be noted that the phase angle between the in-line and transverse motion (θ) is given by typically: $\theta = 230-270°$ (or $-130°$ to $-90°$), within the upper branch regime.

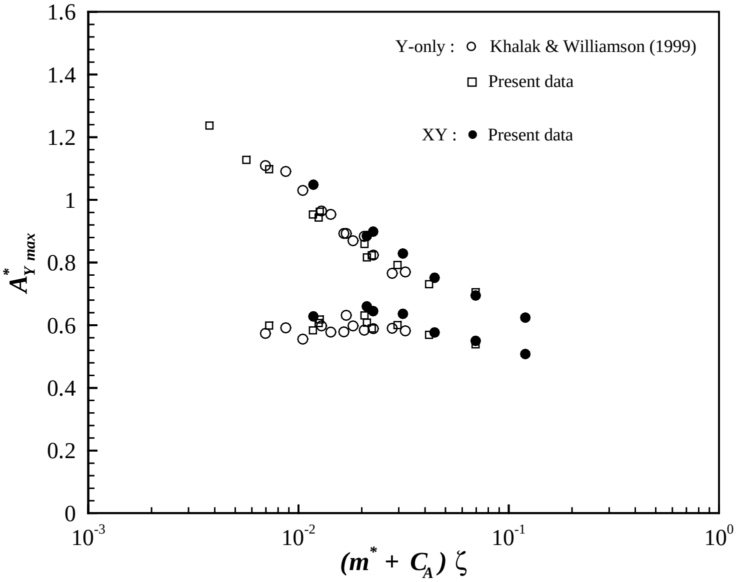

Fig. 2. 'Griffin' plot showing the variation of peak transverse amplitude ($A^*_{Y\,\max}$) with combined mass-damping parameter $[(m^* + C_A)\zeta]$.

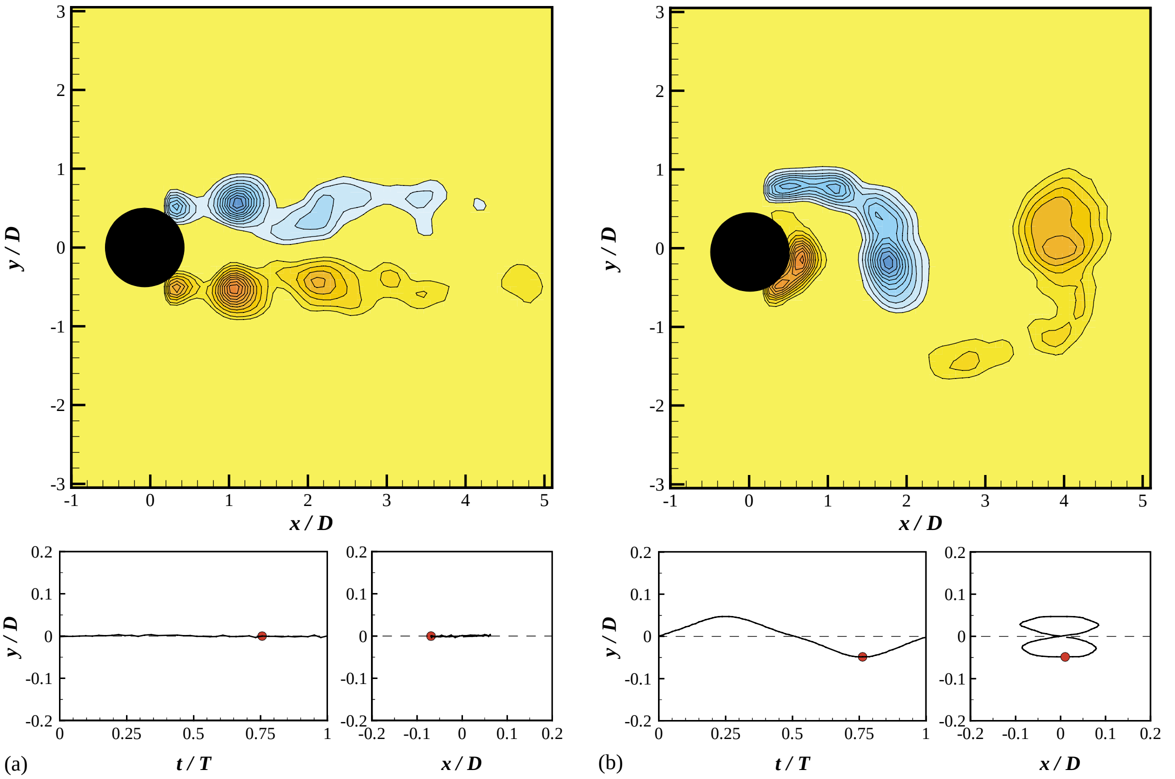

Fig. 3. Vorticity contours, with the corresponding trajectories of body motion, for the two in-line vibration modes. Contour levels are $\{\omega D/U = \pm 0.6, \pm 1.0, \pm 1.4, \ldots\}$.

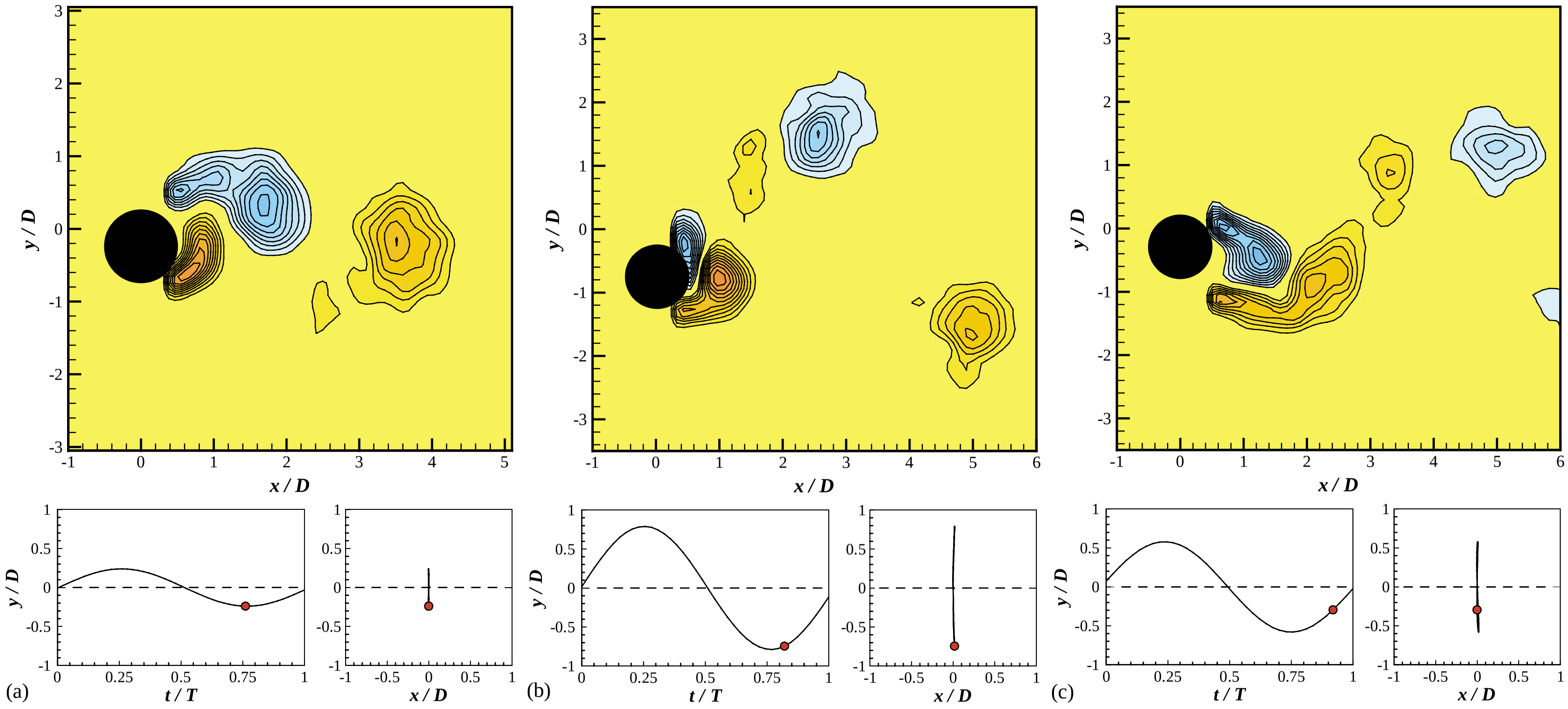

Fig. 4. Vortex formation modes for XY body dynamics are remarkably similar to the vortex dynamics for Y-only motion. (a) Initial branch corresponds to the 2S mode; (b) upper branch corresponds to the 2P mode; (c) lower branch corresponds to the 2P mode. Contour levels are $\{\omega D/U = \pm 0.6, \pm 1.0, \pm 1.4\ldots\}$.

4. Concluding remarks

Despite the large number of papers dedicated to the problem of a cylinder vibrating transverse to a fluid flow (Y motion), there are very few papers which allow the body to vibrate in-line with the flow, as well as transverse to the free stream. Surprisingly, to our knowledge, there are no papers which address the most practical problem, namely a body in two degrees of freedom (XY motion) where the oscillating mass is the same in the transverse and in-line directions, and where the natural frequency is precisely the same in both directions. For this purpose, we have designed the present pendulum apparatus to achieve both of these criteria. An alternative system, with equal mass and frequency in two directions, is the air-bearing platform apparatus built by Prof. Donald Rockwell's group at Lehigh, and results are presently forthcoming from those experiments as well. From all of these experiments, one of the principal questions which may be posed is: *How does the freedom to vibrate in-line with the flow influence the transverse vibration*?

It has been suggested in previous works that dramatic changes in the vortex shedding, forces and responses should be expected, when the body is also free to vibrate streamwise with the flow. However, in the present experiments, it is remarkable that the freedom to oscillate in-line with the flow affects the transverse vibration surprisingly little. This holds true down to the low mass ratios of at least $m^* = 5$. It should also be mentioned that the response exhibits distinct response branches and wake modes, rather than continuous response amplitude plots, as observed in previous XY-motion studies.

In the present case of an elastically mounted cylinder, two in-line vibration modes are found for velocities below the regime of significant transverse vibration, when $U^* \sim 1/2S \sim 2.5$. These two modes are not unexpected, and are equivalent to the first and second excitation modes of vibration for flexible cantilevers (and full-scale piles) in a current, discovered by Wooton et al. (1972), and King (1974), and also observed later by Ongoren and Rockwell (1988) in experiments where they forced bodies to vibrate in-line with the flow.

In conclusion, one might have expected at the start of this study that the freedom of the body to vibrate streamwise would have markedly modified the response dynamics in vortex-induced vibration, and it is surprising that even over a regime of low mass ratios ($m^* > 5$), the transverse response and modes are only almost unaffected. Therefore, a significant conclusion from this study might be that the extensive studies over the past several decades, concerning the case where a body is restrained to move only laterally to the fluid flow, remain relevant and valid for the problem of vortex-induced vibration of a body in two degrees of freedom. The question as to whether this holds true for even smaller mass ratios, even down to $m^* \sim 1$, is presently being addressed in our laboratories.

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